PROCEEDINGS OF THE 2nd WORKSHOP ON POSITIONING, NAVIGATION AND COMMUNICATION (WPNC'05)
& 1st ULTRA-WIDEBAND EXPERT TALK (UET'05)

Model Supported Localization in Cellular Radio Systems

Wolfgang Wilhelmi[a]

[a] IBSe GmbH, 10369 Berlin, Germany, e-mail: w.wilhelmi@ibse-gmbh.de

Abstract - The paper presents a generalized approach to the enhanced localization of moving objects within the framework of localization services as provided by wireless operators. The approach is based on 4 components.
(1) The object moves along a deterministic path representing the skeleton of its random walks. A random walk will be defined by a linear stochastic difference equation.
(2) The position determination procedure of the radio network yields a set of data which can be translated to the parameters of the probability density of the object position as seen by the location service.
(3) An optimal fix of the object is derived from the known density distributions by a type of Maximum Likelihood estimation.
(4) The before mentioned local fixes allow the backtracking of the object state to an intermediate state using a least square error criterion.

1 Introduction to the problem

The most common location method GPS will be augmented or even partially replaced by wireless cellular networks. One advantage of cellular networks is the possibility to use the mobile activity for central localization. The different approaches are presented in the GSM context in [1]. The position precision is limited to ranges from 5 to 200 m, and novel methods are required to refine the fixes. Three types of augmentation techniques can be distinguished:

- Collect additional signals, e.g. from other base stations or special equipment, use proper confidence weighting, and make statistical based evaluations. [2] gives an example.
- Use learning methods to regard experiences of former fixes and to correlate a rich data set. The publication [3] use artificial neuronal networks.
- The dead reckoning approach (e.g. [4]) makes a guess about the track of the mobile station to enhance the next fixes.

The presented paper tries to extend the dead reckoning method to a dynamic model.

As in [1] MLC means the Mobile Location Center; further we assume that the MLC processing unit transacts the Location Center Application Protocol (LCAP) for each mobile station. In the sequel mobile station well be called the *object*.

An antenna attached to a moving object will be located at discrete equidistant moments t_i; $i = 0, 1, \ldots$; $t_{i+1} - t_i = \Delta t > 0$ with a certain error with one of the before mentioned techniques. After a delay τ (assumed to be constant) a primary estimate of the locus and its dispersion is available. The delay includes also the data processing time.

Additional information about the kinematic and dynamical parameters of the object to be located is available. It should be used to enhance navigational services. Particularly we have to answer

- how one can reduce the locational error in regard to the locus and if required to the velocity by modeling of the object,
- whether and under which condition a prediction of locus and velocity is possible. This is especially useful to bridge the time gap given by the delay τ.

The general schema of the presented approach is given by figure 1.

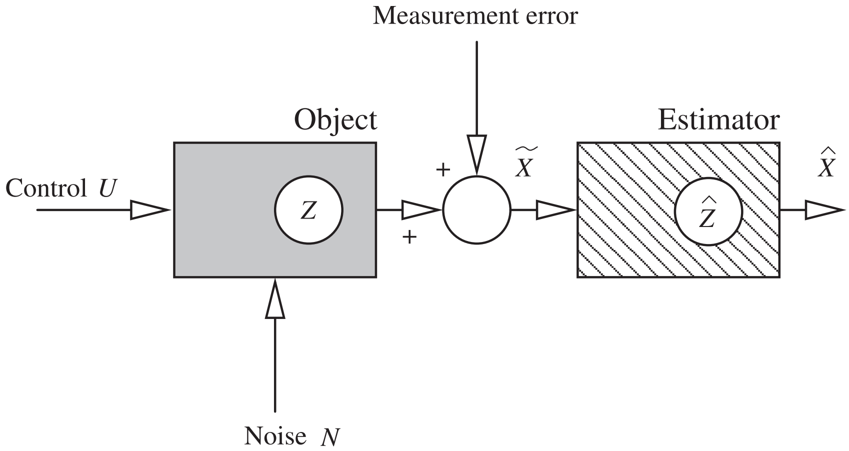

Figure 1: General schema

The object will be described by a time dependent state vector Z, where the locus X and if relevant the velocity $\frac{dX}{dt} = \dot{X}$ are projections of Z. The object is being driven by a control vector U and noise N.The locus vector X with the 3D coordinates x, y, z will be measured by the navigational service [1]. This measurement $\tilde{X}$ is accompanied by explicit or implicit assumption over the error probability. The estimator block should yield an optimal estimate $\hat{X}$ of the actual locus, which relies on an internal estimate $\hat{Z}$ of the object state. Because the state is developed from the past $\hat{Z}$ contains essential parts of the object movement's history. So one can consider the estimator as a dead reckoning instrument.

2 Object model

2.1 General approach

The proposed solution are based on the following assumptions:

- the deterministic control vector is unknown. We subsume its effect as a reference track and set $U \equiv 0$, so driving the object only with N;
- the object moves along a given trajectory (reference track) given as a parametric curve;
- the object obeys a system of linear difference equation of order m [2];
- the probability densities of driving noise N and navigational errors are of the Gaussian type; theoretical and experimental work justify this hypothesis;k=1,. . . ,n

[1]Certain services deliver independent velocity estimates by Doppler effect measurements yielding additional measurement components.

[2]Note that every linear continuous system will be transformed to an equivalent linear difference system by equidistant sampling.

- the problem is treated as a two-dimensional one in the Cartesian coordinates x, y, whereas the extension to the 3D case is easy.

2.2 Reference trajectory

The deterministic 2D reference track is a open or closed curve in the plane.

$$\zeta(t) = \begin{pmatrix} \xi(t - t_s) \\ \eta(t - t_s) \end{pmatrix} \tag{1}$$

Usually this trajectory represents the ideal locus of the mobile object at any time t and is defined by the centerline of lanes or sidewalks. Indeed due to the parameter t_s a set of trajectories with different start time is being defined. t_s will be considered as one of the parameters subject to adaptation. With t_s given sampling in the discrete times t_i yields a sequence of state vector instances $\zeta(i)$.

2.3 Object state and output equation

The state vector has the dimension $m > 2$ with the components $z_j(t);\ j = 1, \ldots, m$. The dimension will correspond to the order of the difference equation and represents an essential model parameter.

$$Z = \begin{pmatrix} z_1 \\ \cdots \\ z_n \end{pmatrix} \tag{2}$$

The observable Cartesian coordinates of the object are given by the output equation:

$$\begin{pmatrix} x - \xi \\ y - \eta \end{pmatrix} = C \cdot Z \quad \text{or more simple} \quad X = C \cdot Z \tag{3}$$

The observation matrix C has the format$2 \times n$. Further $rank(C) = 2$ is required to avoid algebraic dependency between the coordinates.

In the case of independent velocity estimation the observation matrix has the format $4 \times m$ and should satisfy $rank(C) = 4$.

2.4 State equation

The dynamic of the object will be modeled by m first order difference equation driven by scalar white noise.

$$Z(t_{i+1}) = A(t_i) \times Z(t_i) + B(t_i) \times s(t_i) \tag{4}$$

At this mean

- A the system matrix with format $m \times m$,
- B the control matrix with format $m \times 1$, indeed a control vector.
- s a scalar random sequence (noise). its members are stochastic independent random variables, i.e.:

$$\mathbf{E}[s_i \times s_j] = 0; i \neq j \tag{5}$$

They have the distribution $F_s(\cdot)$ with vanishing expectation value μ :

$$\mu = \mathbf{E}[s] = \int_{-\infty}^{-\infty} x \, dF_s(x) = 0 \tag{6}$$

The dispersion is σ^2 is assumed to be 1:

$$\sigma^2 = \mathbf{E}[s \times s] = \int_{-\infty}^{-\infty} x^2 \, dF_s(x) = 1 \tag{7}$$

If system and control matrices do not depend on time, the system is stationary. However, this condition is not necessary for the following considerations.

2.5 Stochastic description

2.5.1 State expectation and covariance

The following concepts can be found and verified in any textbook about causal stochastic systems, e.g. [5]. Under the pre-conditions given in section 2.3 obeys the expectation $\mathcal{Z} = \mathbf{E}[Z]$ of the state Z the following difference equation.

$$\mathcal{Z}(i+1) = A(i) \times \mathcal{Z}(i) \quad \text{with the initial condition} \quad \mathcal{Z}(0) = Z_0 \tag{8}$$

The initial state $Z(0)$ itself is a random vector variable.

The covariance matrix $\mathcal{K}$ of the random vector variable is defined as the expectation of its dyadic product.

$$\mathcal{K}(i,j) = \mathbf{E}[Z(i) \times Z(j)^\top] \tag{9}$$

For $j = i$ and with $\mathcal{K}(i,i) \longrightarrow \mathcal{K}(i)$ one obtains a similar difference equation:

$$\mathcal{K}(i+1) = A(i)\mathcal{K}(i)A^\top(i) + B(i)B^\top(i) \tag{10}$$

As initial condition the covariance matrix $\mathcal{K}(0) = K_0$ of the initial vector $Z(0)$ has to be used. The deterministic case can be treated by

$$K_0 = \varepsilon I \quad \text{with} \quad 0 < \varepsilon \ll 1. \tag{11}$$

I is the m-dimensional unity matrix.

Expectation $\mathcal{X} = \mathbf{E}[X]$ and covariance $\mathcal{L} = \mathbf{E}[X \times X^\top]$ of the observable vector X can be easily found as

$$\mathcal{X} = C \times \mathcal{Z} \tag{12}$$

$$\mathcal{L} = C \times \mathcal{K} \times C^\top. \tag{13}$$

2.5.2 Distribution density

It is well known that Gaussian noise and Gauss distributed initial states yield Gauss distributed states Z and outputs X. Even if this assumption fails their distributions approximate the Gauss distribution after a sufficient number of recurrence steps. In this case expectation and covariance define completely all probability density functions (PDF) with arbitrary dimension.

Let us consider the PDF of X.

$$p_C(X) = \frac{1}{2\pi\sqrt{\det(\mathcal{L})}} \exp\big(-\frac{1}{2}(X^\top - \mathcal{X}^\top)\mathcal{L}^{-1}(X - \mathcal{X})\big) \tag{14}$$

The covariance matrix are given by:

$$\mathcal{L} = \begin{pmatrix} a & c \\ c & b \end{pmatrix} \tag{15}$$

Beyond the symmetry it is positive definite (15):

$$0 < a, \quad 0 < b, \quad c^2 < ab \tag{16}$$

2.5.3 Geometric interpretation

The contour of constant probability density p_r of equation (14 is an ellipse $\mathcal{E}_r$ (see figure 2. The probability of the object's being in the inner of $\mathcal{E}_r$ is given by:

$$P_r = \mathbf{P}[X \in \mathcal{E}_r] = 1 - 2\pi\sqrt{\det(\mathcal{L})} \cdot p_r = 1 - \frac{p_r}{p_{max}} \tag{17}$$

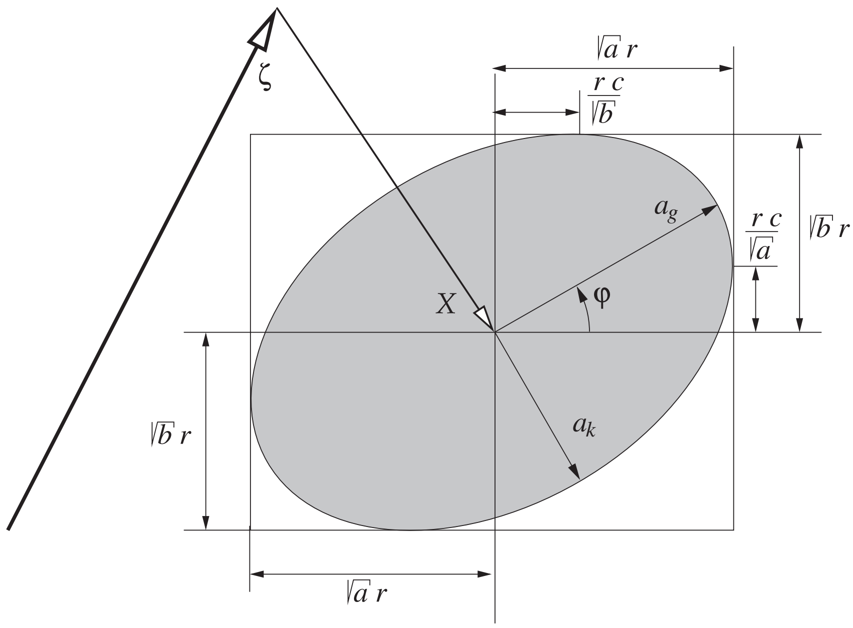

Figure 2: Probability density contour

Later we need the following equations linking the ellipse parameters half-axes and slant with the covariance matrix.

$$a = \frac{(a_g^2 + a_k^2) + (a_g^2 - a_k^2)\cos(2\varphi)}{-4\ln(1 - P_r)} \tag{18}$$

$$b = \frac{(a_g^2 + a_k^2) - (a_g^2 - a_k^2)\cos(2\varphi)}{-4\ln(1 - P_r)} \tag{19}$$

$$c = \frac{(a_g^2 - a_k^2)\sin(2\varphi)}{-4\ln(1 - P_r)}. \tag{20}$$

2.6 System design

We use a certain specialization of the system equations.

- The observation matrix doesn't depend on the time.

$$C = \begin{pmatrix} 1 & q & 0 & \cdots & 0 \\ -1 & q & 0 & \cdots & 0 \end{pmatrix} \quad (21)$$

  with $q > 0$.

- The control matrix depends on time, or more precisely on the sample index i by an varying factor $g_i > 0$.

$$B^{\top}(i) = g_i(0, 0, \cdots, 0, 1) \quad (22)$$

- The system matrix be time dependent by a scalar relaxation coefficient $c_i > 0$, describing a model inherent feedback.

$$A(i) = I + h_i \cdot F \quad (23)$$

  F constitutes a Frobenius matrix of the following form.

$$F = \begin{pmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ -f_1 & -f_2 & -f_3 & -f_4 & \cdots & -f_n \end{pmatrix} \quad (24)$$

  Let be the matrix elements $f_j (j = 1, \dots, n)$ the coefficients of a polynom in λ:

$$f_1 + f_2\lambda + f_3\lambda^2 + f_4\lambda^3 + \cdots + f_n\lambda^{n-1} + \lambda^n = (\lambda - \lambda_1)(1 - \lambda_2)\cdots(\lambda - \lambda_n).$$

  Then the roots λ_k have to satisfy the stability condition.

$$|1 + \max(h_i)\lambda_k| < 1; \quad k = 1, \dots, n$$

These rules assure a stable low pass system in respect to the white noise input. The stability together with the special forms of the control and observation matrices guarantee the resolvability and robustness of the estimation algorithms described in the following section. In practice the system design starts with certain stable configurations of roots, depending on 2 or 3 parameters. Together with h, g, q they are adapted to different types of vehicles. This identification procedure is out of scope of this paper [3].

3 Measurement model

3.1 Coordinates and error distribution

We assume the following correspondence (table 1) between the geographic coordinates and the tangential plane coordinates as used in this paper.

x_0, y_0 mean arbitrary reference coordinates, R_T the radius of earth, and Φ_m the average latitude of the considered path. We don't consider these transformations further because they are static an can be incorporated to the algorithms as simple mappings.

[3] It is worth noting that this task can be done similar to the LSE method presented in section 4.2 either offline or online.

Parameter	LCAP content	Current paper
Latitude	Φ	$y = y_0 + R_T\Phi$
Longitude	Λ	$x = x_0 + R_T \cos\Phi_m\Lambda$
Heading	Θ	$\varphi = \frac{\pi}{2} - \Theta$

Table 1: Coordinate correspondence

2D Gauss distribution A MLC delivers at a certain time t_i a fix specified as an ellipse $\mathcal{E}_s$ as function of P_s

$$P_s = \mathbf{P}[X \in \mathcal{E}_s]$$

with center at

$$\tilde{X} = \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix}.$$

The slant angle and the half-axes of $\mathcal{E}_s$ allow the specification of an equivalent covariance matrix $\mathcal{M}$ by using the formulas (18) after replacing P_r by P_s.

So the PDF of a single fix is given as

$$p_M(X) = \frac{1}{2\pi\sqrt{\det(\mathcal{M})}} \exp\big(-\frac{1}{2}(X^\top - \tilde{X}^\top)\mathcal{M}^{-1}(X - \tilde{X})\big). \tag{25}$$

R, Θ separable chopped Gauss distribution Optionally the MLC provides a fix consisting of independent measurements in polar coordinates. Each measurement is characterized by a chopped 1D Gauss distribution. The must likely locus is given by

$$\tilde{X} = \frac{R_g + R_k}{2}\begin{pmatrix} \sin(\frac{\Theta_g+\Theta_k}{2}) \\ \cos(\frac{\Theta_g+\Theta_k}{2}) \end{pmatrix} + X_R = R_\mu\begin{pmatrix} \sin(\Theta_\mu) \\ \cos(\Theta_\mu) \end{pmatrix} + X_R. \tag{26}$$

$R_g > R_k > 0$ and $\Theta_g > \Theta_k \geq 0$ are the offsets to the reference point X_R as specified in the LCAP.

$$p_M(R, \Theta) = \frac{1}{\pi ab} \times \frac{1}{\big(1 + \mathrm{erf}(\frac{R_\mu}{\sqrt{2a}})\big)\mathrm{erf}(\frac{\pi}{\sqrt{2b}})} \exp\big(-\frac{(R - R_\mu)^2}{2a} - \frac{(\Theta - \Theta_\mu)^2}{2b}\big) \tag{27}$$

Regarding (26) equation (27) yields the joint PDF of the locus. Obviously the R, Θ covariance matrix has diagonal form ($c = 0$). The contour of constant probability density is a standard ellipse $\mathcal{E}_t$ in R, Θ [4] with the center R_μ, Θ_μ and the half axes $(R_g - R_k)/2$, $(\Theta_g - \Theta_k)/2$. Let be

$$P_t = \mathbf{P}[X \in \mathcal{E}_t].$$

the probability of the locus being within $\mathcal{E}_t$. Then the parameters a, b can be computed as follows.

$$a = \frac{(R_g - R_k)^2}{-8\ln(1 - P_t)}; \quad b = \frac{(\Theta_g - \Theta_k)^2}{-8\ln(1 - P_t)} \tag{28}$$

[4]But not in the Cartesian coordinates.

4 Estimator design

4.1 Local fix enhancement

Any local fix is being enhanced by considering the expectation of the locus and the single measurement at the moment t_i only. Information from preceding fixes doesn't care.

The object dynamic allow the hypothesis that it is in the surrounding dX of X with the probability $p_C(X)\,dX$. The localization service asserts that this hypothesis is compatible to its estimation with the probability $p_C \cdot p_M(X)\,dX$. An optimal decision can be based on the following expression.

$$\begin{aligned} L(X) &= p_C(X) \cdot p_M(X) \rightarrow \max_X \\ &\text{oder} \\ \left.\frac{\partial \ln L}{\partial X}\right|_{(X=\hat{X})} &= 0 \end{aligned} \tag{29}$$

An decision should be rejected if the optimum is below a certain fixed threshold.

$$L(\hat{X}) \geq L_l \tag{30}$$

2D Gauss distribution of measurement (case 1) In this case a direct solution (29) is available. Let us use the designations of table 2.

	Object	Measurement
Locus expectation	$\begin{pmatrix} x_m \\ y_m \end{pmatrix}$	$\begin{pmatrix} x_l \\ y_l \end{pmatrix}$
Covariance matrix	$\begin{pmatrix} a_m & c_m \\ c_m & b_m \end{pmatrix}$	$\begin{pmatrix} a_l & c_l \\ c_l & b_l \end{pmatrix}$

Table 2: Decision parameters for case 1

With 2D Gauss distribution

$$N_{ml} = (a_m + a_l)(b_m + b_l) - (c_m + c_l)^2$$

the optimal local fix is given by:

$$\begin{aligned} \hat{x} &= \frac{1}{N_{ml}}\big((a_l(b_m + b_l) - c_l(c_m + c_l))x_m + (a_m(b_m + b_l) - c_m(c_m + c_l))x_l \\ &+ (a_m c_l - a_l c_m)y_m + (a_l c_m - a_m c_l)y_l\big) \end{aligned} \tag{31}$$

$$\begin{aligned} \hat{y} &= \frac{1}{N_{ml}}\big((b_m c_l - b_l c_m)x_m + (b_l c_m - b_m c_l)x_l \\ &+ (b_l(a_m + a_l) - c_l(c_l + c_m))y_m + (b_m(a_m + a_l) - c_m(c_l + c_m))y_l\big) \end{aligned} \tag{32}$$

R, Θ separable chopped Gauss distribution (case 2) There is no explicit solution for equation (29). In the space of the polar coordinates R, Θ one obtains 2 nonlinear equations for $\hat{R}, \hat{\Theta}$ which cannot presented here due to lack of space. These roots have to be found by appropriate numerical methods. Newton's method or successive approximation have been proved successfully [6].

4.2 State backtracking and time shift

Firstly we assume that no time shift occurred, i.e. the t_s is held constant. A sequence of local enhanced fixes at different times allow to refine the object movement. The proposed approach derives an improved estimate of the initial state $\hat{Z}_0$ or more general a state several time intervals ago.

Let be Z_0 an unknown initial state. The valid local fixes satisfying the condition (30) define a sequence of sample indices $\mathbf{F}(m)$. It should contain at least m (the system's order) indices. With the weight matrix

$$G(i,j) = \prod_{k=j}^{i-1} A(k) \tag{33}$$

the observable system at time sample t_i output is given by

$$\check{X}(i) = CG(i,0)Z_0 + C\sum_{j=0}^{i-1} G(i,j)B(j) \cdot s(j). \tag{34}$$

We apply the least square (LS) criterion to derive an LS estimate of Z_0:

$$Q(Z_0) = \mathbf{E}[\sum_{i \in \mathbf{F}(m)} (\check{X}(Z_0) - \hat{X})^\top (\check{X}(Z_0) - \hat{X})] \rightarrow \min_{Z_0} \tag{35}$$

Using the suppositions for $s(j)$, the equations (5) and (7), the optimum is found by the solution of the following symmetric and positive definite linear equation system. Cholesky's method is recommended for this task [6].

$$\mathcal{V} = \mathcal{W} \cdot \hat{Z}_0 \quad \text{with} \tag{36}$$

$$\mathcal{V} = \sum_{i \in \mathbf{F}(m)} G^\top(i,0) C^\top \hat{X}(i) \quad \text{and} \tag{37}$$

$$\mathcal{W} = \sum_{i \in \mathbf{F}(m)} G^\top(i,0) C^\top C G(i,0) \tag{38}$$

Up to now the possible deviations from the observed object track are assumed to be due an inaccurate initial state. Another reason for deviations may be inaccurate timing, usually by speed drift. For this the time shift t_s in equation (1) will be considered as variable in the optimization problem (35). Different values of t_s result in different estimation vectors $\hat{X}(i)$ in equation (36). A search procedure starting with the actual value t_s consists of the iterative solution of (36). There are no practical experiences beyond the simple scanning of a reasonable interval.

4.3 State update and prediction

The consideration of section 4.2 we be connected with a sliding sample window. The window consist of the most recent m samples of valid local fixes. The initial state Z_0 will be identified with the state Z_{i-m}. Starting with an enhanced estimate equations (8) and (10) are used to estimate locus expectation and covariance. These parameters allow the local enhancement of fixes as described in section 4.1. By using (36) one obtains a better estimate of Z_{i-m} and t_s. The optimal estimates $\hat{Z}_{i-m}$ and $\hat{t_s}$ are accompanied by a new covariance matrix assuming the state $\hat{Z}_{i-m}$ as almost deterministic.

$$\hat{\mathcal{K}}_{i-m} = \varepsilon I \quad \text{with} \quad \varepsilon \approx \frac{\Delta^2}{10} \tag{39}$$

Δ should not exceed the required accuracy.

Starting from this the whole computation will be repeated. [5]

When the MLC delivers a new positioning message, the window is shifted ($i := i + 1$) and the enhancement process starts again with the

$$Z_{i-m-1} := \hat{Z}_{i-m-1} \tag{40}$$

and

$$\mathcal{K}_{i-m-1} = \hat{\mathcal{K}}_{i-m-1}. \tag{41}$$

For a prediction one uses the most recent index window and develop the state expectation following equation (8) together with the output equation (12). For intermediate times an interpolation approach is feasible.

5 Conclusion

The presented approach to enhanced location services can be considered as a generalization to dead reckoning methods. It is based on well known mathematical foundations and seems applicable to a broad spectrum of applications. Appropriate numerical methods are recommended too. The adaptation to real world problems has to be done by identification of system parameters and was not considered in this paper. The feasibility was proved with a simulation tool, where the MLC fixes have been modeled by thinning of Poisson point processes. The algorithms are being implemented and proved together with an application project partner [7].

References

[1] ETSI Document "Digital cellular telecommunications systems – Location Services (LCS)" *ETSI TS 101 7124 V7.2.1*, 2000

[2] H.L.Song "Automatic Vehicle Location in Cellular Systems" *IEEE Trans. on Vehicular Technology*, vol.43, no.4, pp. 902-908, 1994

[3] Z.Salcic and E.Chan "Mobile Station Positioning Using GSM Cellular Phone and Artificial Neural Networks" *Wireless Personal Communications* vol.14, no.13, pp. 235-254

[4] M.Bent, M.A.Spirito, and E.Turco "Method for Positioning GSM Mobile Stations Using Absolute Time Delay Measurements" *Electronic Letters*, vol.33, no.33, pp. 2019-2020, 1997

[5] V.S.Pugačev and I.N.Sinizyn, "Stochastičeskiye differencialnye sistemy" *NAUKA Publishing*, Moscow 1985

[6] B.P.Demidovich and I.A.Maron, "Computational Mathematics" *MIR Publishers*, Moscow 1987

[7] INTERVISTA "`http://www.myintervista.de:8060/icosweb/help.htm`", D-14467 Potsdam

[5] Further iterations has been proved as not feasible because due to the window shift every valid fix is treated m times.